Exercise 1

1. The truth table for the given Boolean function is:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

The pseudocode given in the lecture notes has to be followed, for the above table. The first step is to figure out, what is the root node, i.e. the node with the highest information gain.

\[
\text{Entropy}(S) = -\frac{1}{4} \log_2(1/4) - \frac{3}{4} \log_2(3/4) = 0.8113 \quad (1)
\]

\[
\text{Gain}(S, A) = E(S) - 2/4 \cdot E(A = 0) - 2/4 \cdot E(A = 1)
\]
\[
= 0.8113 - 2/4 \cdot (-0 - 0) - 2/4 \cdot (-1/2 \cdot \log_2(1/2) - 1/2 \cdot \log_2(1/2))
\]
\[
= 0.3113 \quad (2)
\]

\[
\text{Gain}(S, B) = E(S) - 2/4 \cdot E(B = 0) - 2/4 \cdot E(B = 1)
\]
\[
= 0.8113 - 2/4 \cdot (-1/2 \cdot \log_2(1/2) - 1/2 \cdot \log_2(1/2)) - 2/4 \cdot (-0 - 0)
\]
\[
= 0.3113 \quad (3)
\]

Since both \(A, B\) have the same information gain, either of them can be chosen as the root node, and the decision tree is shown in Figure 1.

2. Apply the same procedure as in 1.
Exercise 2

1. \[ \text{Entropy}(S) = -\frac{3}{6} \cdot \log_2 \frac{3}{6} - \frac{3}{6} \cdot \log_2 \frac{3}{6} = 1 \] (4)

2. \[
\text{Gain}(S, a_2) = E(S) - \frac{4}{6} E(T) - \frac{2}{6} E(F) \\
= 1 - \frac{4}{6} \left( -\frac{2}{4} \log_2 \frac{2}{4} - \frac{2}{4} \log_2 \frac{2}{4} \right) \\
- \frac{2}{6} \left( -\frac{1}{1} \log_2 \frac{1}{1} - \frac{1}{1} \log_2 \frac{1}{1} \right) = 0.3333 \] (5)

Exercise 3

The pseudocode given in the lecture notes has to be followed.

The first step is to figure out, what is the root node, i.e. the node with the highest information gain.

\[ \text{Entropy}(S) = -\frac{3}{4} \log_2 \frac{3}{4} - \frac{1}{4} \log_2 \frac{1}{4} = 0.8113 \] (6)

\[
\text{Gain}(S, \text{Lecturer}) = E(S) - 3/4 E(\text{Good}) - 1/4 E(\text{Average}) \\
= 0.8113 - 3/4 \ast (-3/3 \ast \log_2 (3/3) - 0) - 1/4 \ast (0 - \log_2 (1/1)) \\
= 0.8113 \] (7)

\[
\text{Gain}(S, \text{Difficulty}) = E(S) - 3/4 E(\text{High}) - 1/4 E(\text{Low})
\]
\[ = 0.8113 - \frac{3}{4} \times \left( -\frac{2}{3} \times \log_2(2/3) - \frac{1}{3} \times \log_2(1/3) \right) \]
\[ - \frac{1}{4} \times \left( -\frac{1}{1} \times \log_2(1/1) - 0 \right) \]
\[ = 0.1226 \quad (8) \]

\[ Gain(S, Subject) = E(S) - \frac{2}{4}E(Good) - \frac{2}{4}E(Average) \]
\[ = 0.8113 - \frac{2}{4} \times \left( -\frac{2}{2} \times \log_2(2/2) - 0 \right) \]
\[ - \frac{2}{4} \times \left( -\frac{1}{2} \times \log_2(1/2) - \frac{1}{2} \times \log_2(1/2) \right) \]
\[ = 0.3113 \quad (9) \]

The root node is therefore Lecturer and the whole procedure has to be repeated for the sub-trees of the tree node. The left subtree (Lecturer is good) should be determined by recalculating the corresponding value for a new set Examples which consist of the original data set for which Lecturer = Good (i.e. rows 1, 2 and 4). Similarly for the right subtree the examples should consist of the original data set for which Lecturer = Average (i.e. only row 3).

However, it can be noticed that for Lecturer = Good all the examples are positive, while for Lecturer = Average all the examples are negative, so the procedure does not need to be repeated again and the final decision tree is shown in Figure 2.

![Figure 2: The decision tree for Exercise 3.](image-url)